Delkapittel 8.6

Oppgave 1

Beregn det bestemte integralet tilnærmet ved å benytte rektangelmetoden.

a, $\int_0^1 x^4 \, dx \qquad n=2$

- Siden $n=2$, er $\Delta x = \frac{b-a}{n} = \frac{1-0}{2} = \frac{1}{2}$

- Dette gir $x_0 = 0$, $x_1 = \frac{1}{2}$ og $x_2 = 1$, og $x_1^* = 0{,}25$ og $x_2^* = 0{,}75$

$$\int_0^1 x^4 \, dx \approx \frac{1-0}{2}\left[f(0{,}25) + f(0{,}75)\right]$$

$$\approx \frac{1}{2}\left(0{,}25^4 + 0{,}75^4\right) \approx \underline{0{,}16016}$$

Eksakt:

$$\int_0^1 x^4 \, dx = \left[\frac{1}{5}x^5\right]_0^1 = \frac{1}{5} = 0{,}2$$

b, $\int_0^1 \sqrt[3]{x} \, dx \qquad n=4$

- Siden $n=4$, er $\Delta x = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$
- Dette gir $x_0 = 0$, $x_1 = \frac{1}{4}$, $x_2 = \frac{1}{2}$, $x_3 = \frac{3}{4}$ og $x_4 = 1$

$x_1^* = \frac{1}{8}$, $x_2^* = \frac{3}{8}$, $x_3^* = \frac{5}{8}$ og $x_4^* = \frac{7}{8}$

$$\int_0^1 \sqrt[3]{x} \, dx \approx \frac{1-0}{4}\left[f\left(\tfrac{1}{8}\right) + f\left(\tfrac{3}{8}\right) + f\left(\tfrac{5}{8}\right) + f\left(\tfrac{7}{8}\right)\right]$$

$$\approx \frac{1}{4} \cdot 3{,}03258 \approx \underline{0{,}75814}$$

Eksakt:

$$\int_0^1 x^{\frac{1}{3}} \, dx = \left[\frac{1}{\frac{1}{3}+1} x^{\frac{1}{3}+1}\right]_0^1 = \left[\frac{3}{4}x^{\frac{4}{3}}\right]_0^1$$

$$= \frac{3}{4} = 0{,}75$$

oppgave 1c

$$\int_1^4 \frac{1}{x}\,dx \qquad n=6$$

- Siden $n=6$, er $\Delta x = \frac{b-a}{n} = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2}$
- Dette gir $x_0 = 1$, $x_1 = \frac{3}{2}$, $x_2 = 2$, $x_3 = \frac{5}{2}$, $x_4 = 3$, $x_5 = \frac{7}{2}$

$x_6 = 4$

$x_1^* = \frac{5}{4}$, $x_2^* = \frac{7}{4}$, $x_3^* = \frac{9}{4}$, $x_4^* = \frac{11}{4}$

$x_5^* = \frac{13}{4}$, $x_6^* = \frac{15}{4}$

$$\int_1^4 \frac{1}{x}\,dx \approx \frac{4-1}{6}\left[f\left(\frac{5}{4}\right)+f\left(\frac{7}{4}\right)+f\left(\frac{9}{4}\right)+f\left(\frac{11}{4}\right)+f\left(\frac{13}{4}\right)+f\left(\frac{15}{4}\right)\right]$$

$$\approx \frac{1}{2}\cdot\left(\frac{4}{5}+\frac{4}{7}+\frac{4}{9}+\frac{4}{11}+\frac{4}{13}+\frac{4}{15}\right) \approx \underline{1{,}3769}$$

$$\int_1^4 \frac{1}{x}\,dx = \left[\ln|x|\right]_1^4 = \ln 4 - \ln 1 = \ln(2^2)$$

$$= \underline{2\ln 2}$$

Oppgave 4

- Bestem hvert bestemte integral tilnærmet ved å benytte trapesmetoden

a) $\int_0^1 \sqrt{x^3+1}\,dx \qquad n=4$

$\Delta x = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4} = 0{,}25$

- Siden $n=4$, får vi $x_0=0$, $x_1=0{,}25$, $x_2=0{,}50$ $x_3=0{,}75$ og $x_4=1{,}0$

$$\int_0^1 \sqrt{x^3+1}\,dx \approx \frac{1-0}{2\cdot 4}\left[f(0)+2f(0{,}25)+2f(0{,}5)+2f(0{,}75)+f(1{,}0)\right]$$

$$\approx \frac{1}{8}\cdot 8{,}9359 \approx \underline{\underline{1{,}116993293}}$$

b) $\int_0^1 \sin(x^2)\,dx \qquad n=6$

$\Delta x = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$

Siden $n=6$, får vi $x_0=0$, $x_1=\frac{1}{6}$, $x_2=\frac{2}{6}$, $x_3=\frac{3}{6}=\frac{1}{2}$, $x_4=\frac{4}{6}=\frac{2}{3}$

$x_5=\frac{5}{6}$ og $x_6=\frac{6}{6}=1$

$$\int_0^1 \sin(x^2)\,dx \approx \frac{1-0}{2\cdot 6}\left[f(0)+2f(1/6)+2f(1/3)+2f(\tfrac{1}{2})+2f(\tfrac{2}{3})+2f(5/6)+f(1)\right]$$

$$\approx \frac{1}{12}\cdot 3{,}7534 \approx \underline{\underline{0{,}3127852156}}$$

Oppgave 6

Beregn hvert integral tilnærmet ved å benytte Simpsons metode med 2n delintervaller.

a) $$\int_1^2 \frac{\ln x}{x^2+1}\,dx \qquad n = 4$$

$$\frac{\Delta x}{2} = \frac{b-a}{2n} = \frac{2-1}{8} = \frac{1}{8}$$

- Siden vi har $2 \cdot n = 8$ delintervaller, har vi 4 parabler med

$x_0 = 1{,}0$, $x_1 = \frac{9}{8}$, $x_2 = \frac{10}{8} = \frac{5}{4}$, $x_3 = \frac{11}{8}$, $x_4 = \frac{12}{8} = \frac{3}{2}$, $x_5 = \frac{13}{8}$, $x_6 = \frac{14}{8} = \frac{7}{4}$

$x_7 = \frac{15}{8}$ og $x_8 = \frac{16}{8} = 2{,}0$

$$\int_1^2 \frac{\ln x}{x^2+1}\,dx \approx \frac{2-1}{6\cdot 4}\left[f(1{,}0) + 4f\left(\frac{9}{8}\right) + 2f\left(\frac{5}{4}\right) + 4f\left(\frac{11}{8}\right) + 2f\left(\frac{3}{2}\right) + 4f\left(\frac{13}{8}\right) + 2f\left(\frac{7}{4}\right) + 4f\left(\frac{15}{8}\right) + f(2{,}0)\right]$$

$$\approx \frac{1}{24} \cdot 2{,}57669 \approx \underline{\underline{0{,}1073621995}}$$

b) $$\int_0^{\pi} e^{\sin x}\,dx \qquad n = 5$$

$$\frac{\Delta x}{2} = \frac{b-a}{2n} = \frac{\pi - 0}{10} = \frac{\pi}{10}$$

- Siden vi har $2 \cdot n = 10$ delintervaller, har vi 5 parabler med

$x_0 = 0{,}0$, $x_1 = \frac{\pi}{10}$, $x_2 = \frac{2\pi}{10} = \frac{\pi}{5}$, $x_3 = \frac{3\pi}{10}$, $x_4 = \frac{4\pi}{10} = \frac{2\pi}{5}$, $x_5 = \frac{5\pi}{10} = \frac{\pi}{2}$

$x_6 = \frac{6\pi}{10} = \frac{3\pi}{5}$, $x_7 = \frac{7\pi}{10}$, $x_8 = \frac{8\pi}{10} = \frac{4\pi}{5}$, $x_9 = \frac{9\pi}{10}$, $x_{10} = \frac{10\pi}{10} = \pi$

$$\int_0^{\pi} e^{\sin x}\,dx \approx \frac{\pi - 0}{6\cdot 5}\left[f(0{,}0) + 4f\left(\frac{\pi}{10}\right) + 2f\left(\frac{\pi}{5}\right) + 4f\left(\frac{3\pi}{10}\right) + 2f\left(\frac{2\pi}{5}\right) + 4f\left(\frac{\pi}{2}\right) + 2f\left(\frac{3\pi}{5}\right) + 4f\left(\frac{7\pi}{10}\right) + 2f\left(\frac{4\pi}{5}\right) + 4f\left(\frac{9\pi}{10}\right) + f(\pi)\right]$$

$$\approx \frac{\pi}{30} \cdot 59{,}289168 \approx \underline{\underline{6{,}208747155}}$$

Delkapittel 8.7

Oppgave 1

Undersøk om hvert uegentlige integral konvergerer og beregn isåfall verdien

a) $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx = \int_0^1 x^{-\frac{1}{2}}\,dx$

Siden ikke integranden er definert for $x=0$, integrerer vi fra t til 1 $(t>0)$:

$$\int_t^1 x^{-\frac{1}{2}}\,dx = \left[\frac{1}{-\frac{1}{2}+1}\,x^{-\frac{1}{2}+1}\right]_t^1 = \left[2x^{\frac{1}{2}}\right]_t^1 = 2-2\sqrt{t}$$

$$= \underline{2(1-\sqrt{t})}$$

Så lar vi $t \to 0^+$

$$\lim_{t\to 0^+}\left[2(1-\sqrt{t})\right] = 2$$

$\therefore$ Integralet konvergerer med grenseverdi

$$\underline{\int_0^1 x^{-1/2}\,dx = 2}$$

b) $\displaystyle\int_0^3 \frac{1}{3-x}\,dx$

Siden integranden ikke er definert for $x=3$, integrer vi fra 0 til s $(s<3)$:

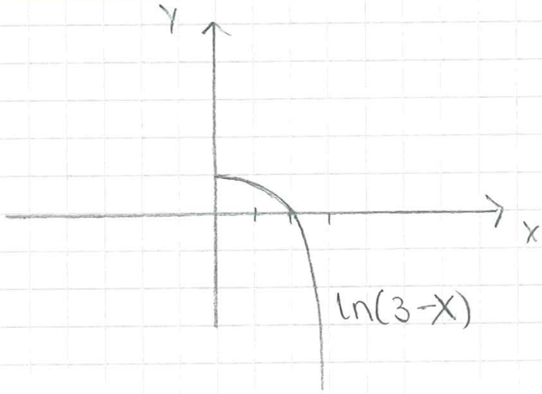
$$\int_0^s \frac{1}{3-x}\,dx = \left[-\ln|3-x|\right]_0^s = -\ln(3-s)+\ln 3$$

$$= \underline{\ln 3 - \ln(3-s)}$$

Så lar vi $s \to 3^-$

$$\lim_{s\to 3^-}\left[\ln 3 - \ln(3-s)\right] = \infty$$

$\therefore$ Integralet divergerer



oppgave 1

c) $$\int_0^2 \frac{1}{\sqrt[3]{2-z}}\,dz$$

Siden integranden ikke er definert for $z=2$, integrerer vi fra 0 til s $(s<2)$:

$$\int_0^s (2-z)^{-\frac{1}{3}}\,dz = \int u^{-\frac{1}{3}} \cdot -du = -\int u^{-\frac{1}{3}}\,du$$

$u(z) = 2-z$

$\frac{du}{dz} = -1$

$dz = -du$

$$= \left[\frac{1}{-\frac{1}{3}+1} u^{-\frac{1}{3}+1}\right] = \left[\frac{3}{2} u^{\frac{2}{3}}\right] = \left[-\frac{3}{2}(2-z)^{\frac{2}{3}}\right]_0^s$$

$$= -\frac{3}{2}(2-s)^{2/3} + \frac{3}{2}\,2^{2/3}$$

$$= -\frac{3}{2}(2-s)^{2/3} + \frac{3}{2}\sqrt[3]{2^2} = \underline{\frac{3}{2}\sqrt[3]{4} - \frac{3}{2}(2-s)^{2/3}}$$

Så lar vi $s \to 2^-$

$$\lim_{s \to 2^-} \left[\frac{3}{2}\sqrt[3]{4} - \frac{3}{2}(2-s)^{2/3}\right] = \frac{3}{2}\sqrt[3]{4}$$

$\supset$: Integralet konvergerer med grenseverdi:

$$\underline{\int_0^2 \frac{1}{\sqrt[3]{2-z}}\,dz = \frac{3}{2}\sqrt[3]{4}}$$

d) $$\int_0^2 \frac{t}{\sqrt{4-t^2}}\,dt$$

Siden integranden ikke er definert for $t=2$, integrerer vi fra 0 til s $(s<2)$:

$$\int_0^s \frac{t}{\sqrt{4-t^2}}\,dt = \int \frac{1}{\sqrt{u}}\, t\left(-\frac{1}{2t}\right)du = \int -\frac{1}{2} u^{-\frac{1}{2}}\,du$$

$u(t) = 4-t^2$

$\frac{du}{dt} = -2t$

$dt = -\frac{1}{2t}\,du$

$$= \left[-\frac{1}{2}\,\frac{1}{-1/2+1}\,u^{-\frac{1}{2}+1}\right] = \left[-u^{\frac{1}{2}}\right] = \left[-\sqrt{4-t^2}\right]_0^s$$

$$= -\sqrt{4-s^2} + \sqrt{4} = \underline{2 - \sqrt{4-s^2}}$$

Så lar vi $s \to 2^-$ $\lim_{s \to 2^-} \left[2 - \sqrt{4-s^2}\right] = 2$

$\supset$: Integralet konvergerer med grenseverdi lik 2