Delkapittel 8.2

oppgave 2

a) $\int_1^2 x^3\,dx = \Big[\frac{1}{4}x^4\Big]_1^2 = \frac{1}{4}\cdot 2^4 - \frac{1}{4}\cdot 1^4$

$= \frac{1}{4}\cdot 16 - \frac{1}{4} = 4 - \frac{1}{4} = \underline{\underline{\frac{15}{4}}}$

b) $\int_{-2}^{2} (x^5 - 2x)\,dx = \int_{-2}^{2} x^5\,dx - \int_{-2}^{2} 2x\,dx$

$= \Big[\frac{1}{6}x^6 - x^2\Big]_{-2}^{2}$

$= \left(\frac{1}{6}\cdot 2^6 - 2^2\right) - \left(\frac{1}{6}(-2)^6 - (-2)^2\right)$

$= \frac{1}{6}\cdot 64 - 4 \quad - \frac{1}{6}\cdot 64 + 4$

$= \underline{\underline{0}}$

c) $\int_{-1}^{0} (t-1)^2\,dt = \int_{-1}^{0} u^2\,du = \Big[\frac{1}{3}u^3\Big]_{-1}^{0}$

$u(t) = t-1$

$\frac{du}{dt} = u'(t) = 1$

$dt = du$

$= \Big[\frac{1}{3}(t-1)^3\Big]_{-1}^{0} = -\frac{1}{3} + \frac{8}{3} = \underline{\underline{\frac{7}{3}}}$

d) $\int_{-\sqrt{2}}^{-1} \frac{3z^2-2}{z}\,dz = \int_{-\sqrt{2}}^{-1} 3z\,dz - 2\int \frac{1}{z}\,dz$

polynomdivision:

$3z^2-2 : z = 3z$

$\underline{3z^2}$

-2

$= \Big[\frac{3}{2}z^2 - 2\ln|z|\Big]_{-\sqrt{2}}^{-1}$

$= \Big[\frac{3}{2}z^2 - \ln|z^2|\Big]_{-\sqrt{2}}^{-1}$

$= \frac{3}{2}\cdot(-1)^2 - \ln(-1)^2 - \frac{3}{2}(-\sqrt{2})^2 + \ln(-\sqrt{2})^2$

$= \frac{3}{2} - 0 - 3 + \ln 2$

$= \underline{\underline{\ln 2 - 3/2}}$

e) $\int_{\pi/6}^{\frac{5\pi}{6}} \sin u\,du = \Big[-\cos u\Big]_{\pi/6}^{5\pi/6}$

$= -\cos\frac{5\pi}{6} - \left(-\cos\frac{\pi}{6}\right)$

$= -\left(-\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \underline{\underline{\sqrt{3}}}$

f) $\int_0^2 \sqrt{9x+2}\,dx = \int_0^2 \sqrt{u}\,\frac{1}{9}\,du = \frac{1}{9}\int_0^2 u^{1/2}\,du$

$u(x) = 9x+2$

$\frac{du}{dx} = 9$

$dx = \frac{1}{9}du$

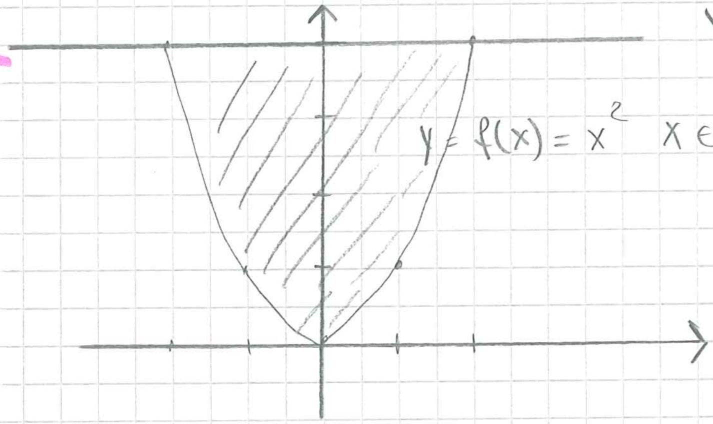
$= \Big[\frac{1}{9}\cdot\frac{2}{3}u^{3/2}\Big]_0^2 = \Big[\frac{2}{27}(9x+2)^{3/2}\Big]_0^2$

$= \frac{2}{27}\sqrt{20^3} - \frac{2}{27}\sqrt{2^3} = \frac{2}{27}\left(40\sqrt{5} - 2\sqrt{2}\right)$

$= \underline{\underline{\frac{4}{27}\left(20\sqrt{5} - \sqrt{2}\right)}}$

Delkapittel 8.3

Oppgave 2



Vi skal bestemme arealet:

Arealet er begrenset øverst av $y=f(x)=4$ og nederst av $y=g(x)=x^2$

Arealet er gitt ved:

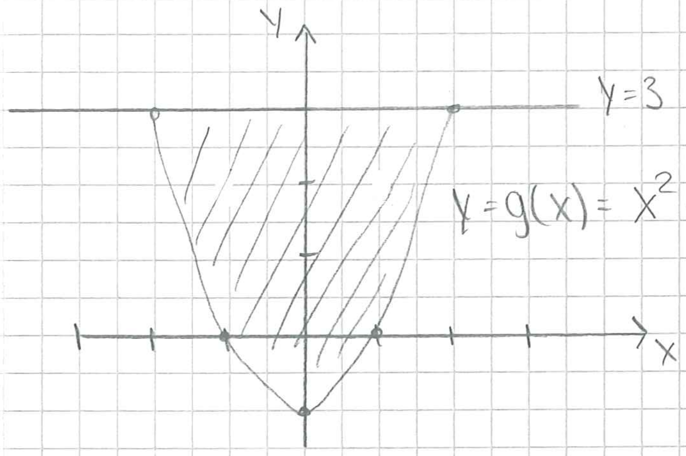
$$A = \int_a^b [f(x) - g(x)]\,dx$$

$$= \int_{-2}^{2} [4 - x^2]\,dx = \left[4x - \frac{1}{3}x^3\right]_{-2}^{2}$$

$$= \left(4 \cdot 2 - \frac{1}{3} \cdot 2^3\right) - \left(4(-2) - \frac{1}{3}(-2)^3\right)$$

$$= 8 - \frac{8}{3} + 8 - \frac{8}{3} = \underline{\underline{\frac{32}{3}}}$$

oppgave 4



Vi skal bestemme arealet:

Arealet er begreset øverst av $y = f(x) = 3$ og nederst av $y = g(x) = x^2 - 1$

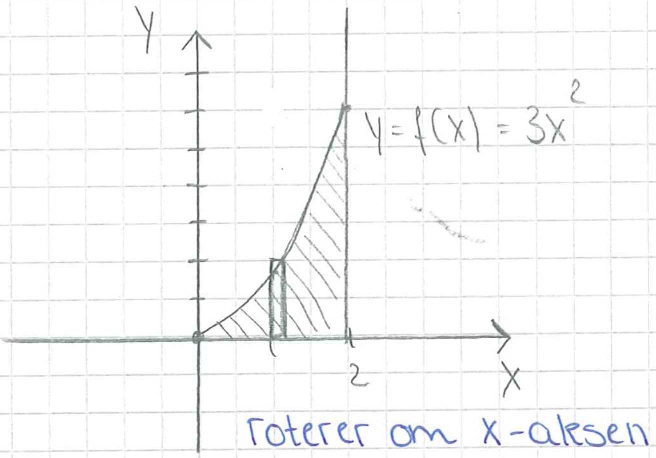
Arealet er gitt ved:

$$A = \int_{-2}^{2} [3 - (x^2 - 1)]\, dx = \int_{-2}^{2} (4 - x^2)\, dx = \left[4x - \frac{1}{3}x^3\right]_{-2}^{2}$$

$$= \underline{\underline{\frac{32}{3}}}$$

Oppgave 8

a)



roterer om x-aksen

Benytter skivemetoden:

- kutter omdreiningslegemet i skiver av plan vinkelrett på x-aksen

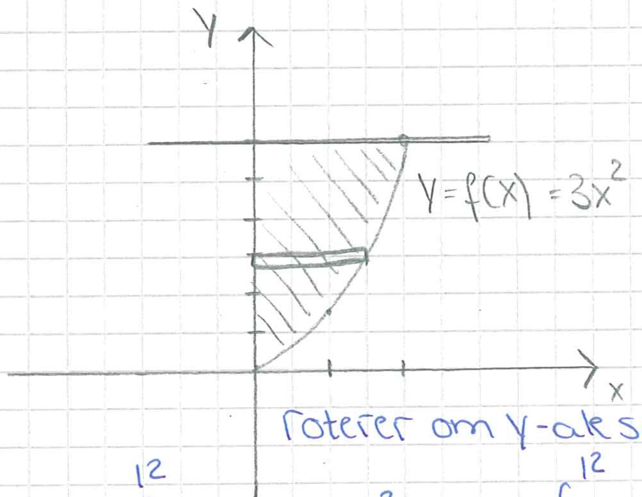
$$V = \int_a^b \pi\,[f(x)]^2\,dx$$

$$= \int_0^2 \pi\,[3x^2]^2\,dx = \pi\int_0^2 9x^4\,dx$$

$$= \pi\left[\frac{9}{5}x^5\right]_0^2 = \pi\cdot\frac{9}{5}\cdot 2^5$$

$$= \underline{\underline{\frac{288\pi}{5}}}$$

b)



roterer om y-aksen

Benytter skivemetoden:

- først må vi finne x uttrykt ved y.

Har at: $y = f(x) = 3x^2$

$$x^2 = \frac{y}{3}$$

$$x = \pm\sqrt{\frac{y}{3}}$$

$$V = \int_0^{12} \pi\left[\sqrt{\frac{y}{3}}\right]^2 dy = \pi\int_0^{12} \frac{y}{3}\,dy$$

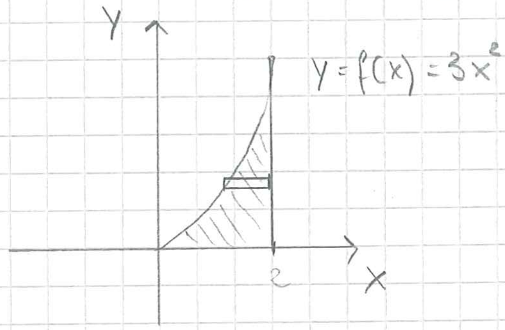
$$= \pi\left[\frac{1}{6}y^2\right]_0^{12} = \pi\cdot\frac{12^2}{6} = \underline{\underline{24\pi}}$$

oppgave 8

a) Benytter sylinderskallmetoden:

$y = 3x^2$

- Vi må integrere mhp y og må finne x uttrykt med y



$x^2 = \frac{y}{3}$

$x = \pm\sqrt{\frac{y}{3}}$

- Lengden er gitt ved: $2 - \sqrt{\frac{y}{3}}$

$$V = \int_0^{12} 2\pi y \left(2 - \sqrt{\frac{y}{3}}\right) dy = 2\pi \int_0^{12} \left(2y - y\sqrt{\frac{y}{3}}\right) dy$$

$$= 2\pi \int_0^{12} \left(2y - \frac{1}{\sqrt{3}} y^{3/2}\right) dy$$

$$= 2\pi \left[y^2 - \frac{1}{\sqrt{3}} \cdot \frac{2}{5} y^{5/2} \right]_0^{12}$$

$$= 2\pi \left[12^2 - \frac{1}{\sqrt{3}} \cdot \frac{2}{5} 12^{5/2} \right] =$$

$$= 2\pi \left[\frac{720}{5} - \frac{576}{5} \right) = 2\pi \cdot \frac{144}{5} = \underline{\underline{\frac{288\pi}{5}}}$$

b) Benytter sylinderskallmetoden:

$y = f(x) = 3x^2$

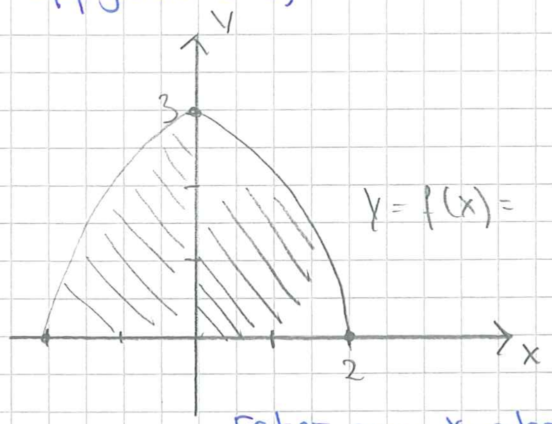
- høyden til sylinderskallet blir $3x^2$

√

$$V = \int_0^2 2\pi x \,(3x^2)\, dx = 2\pi \int_0^2 3x^3\, dx = 2\pi \left[\frac{3}{4} x^4\right]_0^2$$

$$= 2\pi \cdot \frac{3}{4} \cdot 2^4 = 2\pi \cdot 12 = \underline{\underline{24\pi}}$$

Oppgave 8 c)



$y = f(x) = 3\sqrt{1-(x^2/4)}$

roter om x-aksen

Benytter skivemetoden:

$$V = 2\int_0^2 \pi\,[f(x)]^2\,dx = 2\pi\int_0^2 \left[3\sqrt{1-(x^2/4)}\right]^2 dx$$

$$= 18\pi \int_0^2 \left(1 - \frac{x^2}{4}\right) dx$$

$$= 18\pi \left[x - \frac{1}{12}x^3\right]_0^2$$

$$= 18\pi \left[2 - \frac{1}{12}\cdot 8\right] = 18\pi\left[\frac{24}{12} - \frac{8}{12}\right]$$

$$= 18\pi \cdot \frac{16}{12} = \underline{\underline{24\pi}}$$

d) roterer om y-aksen

Benytter skivemetoden:

- først må vi finne x uttrykt med y

Har at $y = 3\sqrt{1-(x^2/4)}$

$$1 - \frac{x^2}{4} = \left(\frac{y}{3}\right)^2$$

$$\frac{x^2}{4} = 1 - \left(\frac{y}{3}\right)^2$$

$$x = \pm\sqrt{4\left(1-\left(\frac{y}{3}\right)^2\right)} = \pm 2\sqrt{1-\left(\frac{y}{3}\right)^2}$$

$$V = \int_0^3 \pi\left[2\sqrt{1-\left(\frac{y}{3}\right)^2}\right]^2 dy = 4\pi\int_0^3\left(1-\left(\frac{y}{3}\right)^2\right)dy = 4\pi\left[y - \frac{1}{9}\cdot\frac{1}{3}y^3\right]_0^3$$

$$= 12\pi - 4\pi = \underline{\underline{8\pi}}$$