Delkapittel 8.1

Oppgave 1

a) $\int (4x^3 - 6x + 1)\,dx$

benytter følgende regel: $\int x^{\lambda} dx = \frac{1}{\lambda+1} x^{\lambda+1} + C$ Tabell 8.1

$$= 4 \cdot \frac{1}{4} x^{3+1} - 6 \cdot \frac{1}{2} x^{1+1} + x + C$$

$$= \underline{\underline{x^4 - 3x^2 + x + C}}$$

b) $\int (x^{15} + 3x^{12} - 5x^4)\,dx$

$$= \underline{\underline{\frac{1}{16} x^{16} + \frac{3}{13} x^{13} - x^5 + C}}$$

c) $\int (x^{-6} + 4x^{1/2} - 7\sqrt[3]{x^2})\,dx$

$x^{\frac{2}{3}}$

$$= -\frac{1}{5} x^{-5} + 4 \cdot \frac{2}{3} x^{3/2} - 7 \cdot \frac{3}{5} x^{5/3} + C$$

$$= \underline{\underline{-\frac{1}{5} x^{-5} + \frac{8}{3} x^{3/2} - \frac{21}{5} x^{5/3} + C}}$$

Oppgave 1

d) $\int \left(\frac{1}{t^4} + \frac{5}{t} - \frac{2}{\sqrt[7]{t^3}} \right) dt$

$= \int \left(t^{-4} + 5t^{-1} - 2 \cdot t^{-\frac{3}{7}} \right) dt$

$= -\frac{1}{3} t^{-3} + 5 \ln t - 2 \cdot \frac{7}{4} t^{\frac{4}{7}} + C$

$= \underline{\underline{-\frac{1}{3} t^{-3} + 5 \ln t - \frac{7}{2} t^{\frac{4}{7}} + C}}$

e) $\int \left[\frac{z^{5/3} \cdot \sqrt[3]{z}}{\sqrt[6]{z^7}} \right] dz$

$= \int \frac{z^{5/3} \cdot z^{1/3}}{z^{7/6}} dz$

$= \int z^{5/3 + 1/3 - 7/6} dz = \int z^{10/6 + 2/6 - 7/6} dz$

$= \int z^{5/6} dz = \underline{\underline{\frac{6}{11} z^{\frac{11}{6}} + C}}$

Oppgave 2

Løs integralene ved bruk av substitusjon

$$\int f(u(x))\,u'(x)\,dx = \int f(u)\,du \qquad \text{Teorem 7.4}$$

a) $\int (5x+2)^3\,dx$

$\frac{du}{dx} = u(x) = 5x+2$

$\frac{du}{dx} = u'(x) = 5 \qquad du = 5\,dx$

$f(u(x))\,u'(x)\,dx = u^3 \cdot 5\,dx$

$f(u)\,du = u^3\,du$

disse skal være like

$du = 5\,dx$

$dx = \frac{1}{5}\,du$

Vi får:

$$\int (5x+2)^3\,dx = \int u^3 \frac{1}{5}\,du$$

$$= \frac{1}{5}\int u^3\,du = \frac{1}{5}\cdot\frac{1}{4}u^4 + C$$

$$= \frac{1}{20}u^4 + C$$

$$= \underline{\underline{\frac{1}{20}(5x+2)^4 + C}}$$

oppgave 2

b) $$\int \frac{dx}{\sqrt[3]{5-7x}} = \int u^{-\frac{1}{3}} \left(-\frac{1}{7}\right) du = -\frac{1}{7} \int u^{-\frac{1}{3}} du$$

$u(x) = 5-7x$

$\frac{du}{dx} = -7$

$dx = -\frac{1}{7} du$

$$= -\frac{1}{7} \frac{3}{2} u^{\frac{2}{3}} + C$$

$$= -\frac{3}{14} (5-7x)^{\frac{2}{3}} + C$$

$$= -\frac{3}{14} \sqrt[3]{(5-7x)^2} + C$$

c) $$\int \frac{dx}{8x+9} = \int \frac{1}{u} \frac{1}{8} du = \frac{1}{8} \int \frac{1}{u} du$$

$u(x) = 8x+9$

$\frac{du}{dx} = u'(x) = 8$

$dx = \frac{1}{8} du$

$$= \frac{1}{8} [\ln u + C$$

$$= \frac{1}{8} \ln(8x+9) + C$$

oppgave 2

d) $\int \cos\left(2u + \frac{\pi}{3}\right) du$

$v(u) = 2u + \frac{\pi}{3}$

$v'(u) = \frac{dv}{du} = 2$

$dv = 2\,du$

$du = \frac{1}{2}\,dv$

Vi har at: Teorem

$$\int f(v(u))\,v'(u)\,du = \int f(v)\,dv$$

$f(v(u))\,v'(u)\,du = \cos v \cdot 2\,du$

$f(v)\,dv = \cos v\,dv$

disse skal altså være like

Dette gir at: $dv = 2\,du$

$du = \frac{1}{2}\,dv$

Vi får:

$$\int \cos\left(2u + \frac{\pi}{3}\right) du = \int \cos v \; \frac{1}{2}\,dv$$

$$= \frac{1}{2} \int \cos v\,dv = \frac{1}{2} \sin v + C$$

$$= \frac{1}{2} \sin\left(2u + \frac{\pi}{3}\right) + C$$

oppgave 3

a) $\int \sin^3 x \, dx = \int (\sin x)^3 \, dx$

$= \int \sin^2 x \, \sin x \, dx = \int (1 - \cos^2 x) \sin x \, dx$

har at:

$\sin^2 x = 1 - \cos^2 x$

setter $u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x \, dx$

$dx = -\frac{1}{\sin x} du$

$= -1 \int (1 - u^2) \, du$

$= -(u - \frac{1}{3} u^3) + C$

$= -\cos x + \frac{1}{3} \cos^3 x + C$

$= \frac{1}{3} \cos^3 x - \cos x + C$

b) $\int \cos^5 x \, dx = \int (\cos x)^5 \, dx$

$= \int (\cos x)^4 \cdot \cos x \, dx = \int (\cos x)^2 (\cos x)^2 \cos x \, dx$

har at:

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$u = \sin x$ $\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$dx = \frac{1}{\cos x} du$

$= \int (1 - \sin^2 x)(1 - \sin^2 x) \cos x \, dx$

$= \int (1 - u^2)(1 - u^2) \, du$

$= \int 1 - 2u^2 + u^4 \, du$

$= u - \frac{2}{3} u^3 + \frac{1}{5} u^5 + C$

$= \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + C$

c) $\int \sin^3 t \cdot \cos^2 t \, dt = \int \sin^2 t \cos^2 t \, \sin t \, dt$

setter:

$u = \cos t \quad \frac{du}{dt} = -\sin t$

$du = -\sin t \, dt$

$$= \int (1-\cos^2 t) \cos^2 t \, \sin t \, dt$$

$$= -1 \int (1-u^2) u^2 \, du$$

$$= -\int u^2 - u^4 \, du$$

$$= -\left[\frac{1}{3} u^3 - \frac{1}{5} u^5 \right] + C$$

$$= \frac{1}{5} u^5 - \frac{1}{3} u^3 + C$$

$$= \underline{\underline{\frac{1}{5} \cos^5 t - \frac{1}{3} \cos^3 t + C}}$$

d) $\int \sin^4 z \cdot \cos^3 z \, dz = \int \sin^4 z \cdot \cos^2 z \, \cos z \, dz$

Setter:

$u = \sin z \quad \frac{du}{dz} = \cos z$

$du = \cos z \, dz$

$$= \int \sin^2 z \cdot \sin^2 z \cdot (1-\sin^2 z) \cos z \, dz$$

$$= \int u^4 \cdot (1-u^2) \, du$$

$$= \frac{1}{5} u^5 - \frac{1}{7} u^7 + C$$

$$= \underline{\underline{\frac{1}{5} \sin^5 z - \frac{1}{7} \sin^7 z + C}}$$

Kap 8.1:

oppgave 4

Delvis integrasjon:

$$\int f(x)\,g'(x)\,dx = f(x)g(x) - \int f'(x)\,g(x)\,dx$$

a) $\int \underset{f(x)}{(2x+1)}\,\underset{g'(x)}{\sin x}\,dx$

$= (2x+1)\cdot(-\cos x) - \int 2(-\cos x)\,dx$

$= -(2x+1)\cos x + \int 2\cos x\,dx$

$= -(2x+1)\cos x + 2\sin x + C$

$= \underline{\underline{-2x\cos x - \cos x + 2\sin x + C}}$

b) $\int \underset{f(x)}{(3x-5)}\,\underset{g'(x)}{\cos x}\,dx$

$(\cos x)' = -\sin x$

$(\sin x)' = \cos x$

$= (3x-5)\sin x - \int 3\sin x\,dx$

$= \underline{\underline{3x\sin x - 5\sin x + 3\cos x + C}}$

c) $\int \underset{f(x)}{(4x+3)}\,\underset{g'(x)}{e^{2x}}\,dx$

$\left(\frac{1}{2}e^{2x}\right)' = \frac{1}{2}(e^u)'u'$

$u = 2x$

$\frac{du}{dx} = u' = 2$

$= \frac{1}{2}e^u \cdot 2$

$= e^u = e^{2x}$

$= (4x+3)\cdot\frac{1}{2}e^{2x} - \int 4\cdot\frac{1}{2}e^{2x}\,dx$

$= 2xe^{2x} + \frac{3}{2}e^{2x} - \int 2e^{2x}\,dx$

$= 2xe^{2x} + \frac{3}{2}e^{2x} - e^{2x} + C$

$= \underline{\underline{\left(2x + \frac{1}{2}\right)e^{2x} + C}}$

d) $\int \ln(x+1)\,dx = \int \underset{f(x)}{\ln(x+1)} \cdot \underset{g'(x)}{1}\,dx$

$= \ln(x+1)\cdot x - \int \frac{1}{x+1}\cdot x\,dx$

$= \ln(x+1)\cdot x - \int \left(1 - \frac{1}{x+1}\right) dx$

$= \ln(x+1)x - \int dx + \int \frac{1}{x+1}\,dx$

$= x\ln(x+1) - x + \ln|x+1| + C$

$= \underline{\underline{(x+1)\ln(x+1) - x + C}}$

polynomdivisjon:

$x : x+1 = 1$
$x+1$

-1

Dette gir:

$\frac{x}{x+1} = 1 - \frac{1}{x+1}$

e) $\int \tan^{-1}x\,dx = \int \underset{f(x)}{\tan^{-1}x} \cdot \underset{g'(x)}{1}\,dx$

$= \tan^{-1}x \cdot x - \int \frac{x}{1+x^2}\,dx$

$u = x^2$

$\frac{du}{dx} = u'(x) = 2x$

$dx = \frac{1}{2x}$

$= x\tan^{-1}x - \int \frac{1}{1+u}\, x \cdot \frac{1}{2x}\,du$

$= x\tan^{-1}x - \frac{1}{2}\int \frac{1}{1+u}\,du$

$= x\tan^{-1}x - \frac{1}{2}\ln|1+u| + C$

$= \underline{\underline{x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + C}}$

f) $\int \sin^{-1}(2x)\,dx = \int \sin^{-1}(2x)\cdot 1\,dx$

$= x\cdot\sin^{-1}(2x) - \int x\cdot\frac{1}{\sqrt{1-(2x)^2}}\cdot 2\,dx$

$= x\sin^{-1}(2x) - \int 2x\cdot\frac{1}{\sqrt{1-4x^2}}\,dx$

$u = 1-4x^2$

$\frac{du}{dx} = -8x$

$dx = \frac{-1}{8x}$

$= x\sin^{-1}(2x) - \int \frac{1}{\sqrt{u}}\,2x\cdot\left(\frac{-1}{8x}\right)du$

$= x\sin^{-1}(2x) + \frac{1}{4}\int u^{-\frac{1}{2}}\,du$

$= x\sin^{-1}(2x) + \frac{1}{4}\cdot 2\,u^{1/2} + C$

$= x\sin^{-1}(2x) + \frac{1}{2}(1-4x^2)^{1/2} + C$

$= x\sin^{-1}(2x) + \frac{1}{2}\sqrt{1-4x^2} + C$

oppgave 11

a) $$\int \frac{x+11}{x^2+2x-3}\,dx = \int \frac{3}{x-1}\,dx - \int \frac{2}{x+3}\,dx$$

Faktoriserer neuner:

$$x^2+2x-3=0$$

når:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4+12}}{2}$$

$$x_1 = \frac{-2+4}{2} = \frac{2}{2} = 1$$

$$x_2 = \frac{-2-4}{2} = \frac{-6}{2} = -3$$

$$= 3\int \frac{1}{x-1}\,dx - 2\int \frac{1}{x+3}\,dx$$

$$= 3\ln|x-1| - 2\ln|x+3| + C$$

$$= \underline{\underline{\ln\left(\frac{(x-1)^3}{(x+3)^2}\right) + C}}$$

Delbrøkoppspaltning:

$$\frac{x+11}{x^2+2x-3} = \frac{A}{(x-1)} + \frac{B}{(x+3)}$$

$$= \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

$$= \frac{(A+B)x + 3A - B}{(x-1)(x+3)}$$

1) $A+B=1$ $\qquad A = 1-B \qquad \underline{\underline{A=3}}$

2) $3A - B = 11$

$3(1-B) - B = 11$

$-3B - B = 11-3$

$-4B = 8$

$\underline{\underline{B=-2}}$

b) $\int \frac{x^2+5x-6}{x^2-2x}\,dx = \int 1\,dx + \int \frac{7x-6}{x^2-2x}\,dx$

Polynomdivisjon:

$x^2+5x-6 : x^2-2x = 1$

x^2-2x

$+7x-6$

delbrøkoppspalt.:

$\frac{7x-6}{x^2-2x} = \frac{A}{x} + \frac{B}{x-2}$

$= \frac{(A+B)x-2A}{x(x-2)}$

1) $A+B=7$ $\quad B=4$

2) $-2A=-6$

$A=3$

$= \int dx + \int \frac{3}{x}\,dx + \int \frac{4}{x-2}\,dx$

$= x + 3\ln|x| + 4\ln|x-2| + C$

$= x + \ln|x^3| + \ln(x-2)^4 + C$

$= \underline{\underline{x + \ln(|x^3|\cdot(x-2)^4) + C}}$

c) $$\int \frac{4x^2+2x+4}{x^3-4x}\,dx = -\int \frac{1}{x}\,dx + 2\int \frac{1}{x+2}\,dx + 3\int \frac{1}{x-2}\,dx$$

Faktoriserer nevner:

$$x^3-4x = x(x^2-4)$$
$$= x(x+2)(x-2)$$

Delbrøkoppspalt.:

$$\frac{4x^2+2x+4}{x^3-4x} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$$

$$= \frac{A(x+2)(x-2) + Bx(x-2) + Cx(x+2)}{N}$$
$$= \frac{A(x^2-4) + B(x^2-2x) + C(x^2+2x)}{N}$$

1) $A+B+C=4$ $\qquad = (A+B+C)x^2 + (2C-2B)x - 4A$

2) $2C-2B=2$

3) $-4A=4$

$A=-1$

1) $B+C=5$

$C=5-B$

2) $2(5-B)-2B=2$

$-4B=2-10$

$B=\frac{-8}{-4}=2$

$C=5-B=5-2=3$

$$= -\ln|x| + 2\ln(x+2) + 3\ln|x-2| + C$$
$$= -\ln|x| + \ln(x+2)^2 + \ln|(x-2)^3| + C$$
$$= \underline{\underline{\ln\left(\frac{(x+2)^2\cdot|(x-2)^3|}{|x|}\right) + C}}$$