Oppgaver kap 4.2

4.2.1.a

Oppgave 1a)

$0 < x < \pi$

$\sin(2x) = \cos x$

$2\sin x \cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0 \qquad x = \frac{\pi}{2}$ eller $x = \frac{3\pi}{2}$

$2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = \frac{1}{2} \qquad x = \frac{\pi}{6}$ eller $x = \frac{5\pi}{6}$

oppgaver kap 4.2

oppgave 1d

$$9\cos(2x) - 5 = 0$$

$$9(\cos^2 x - \sin^2 x) - 5 = 0$$

$$9(1 - \sin^2 x - \sin^2 x) - 5 = 0$$

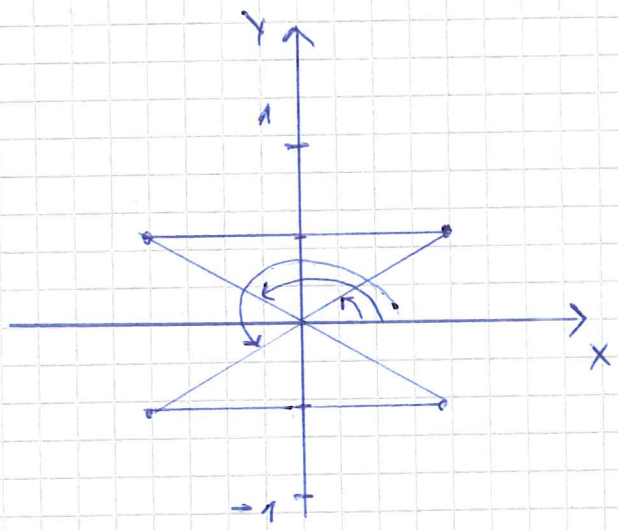
$$9(1 - 2\sin^2 x) - 5 = 0$$

$$9 - 18\sin^2 x - 5 = 0$$

$$18\sin^2 x = 4$$

$$\sin^2 x = \frac{4}{18} = \frac{2}{9}$$

$$\sin x = \pm\sqrt{\frac{2}{9}} = \pm\frac{\sqrt{2}}{3}$$



D: 4 løsninger

$x = 0,49$

$x = 2,65$

$x = 3,63$

$x = 5,79$

(sida 3 finnes ikke i detta dokumentet)

Uppgave 1j

$4\sin(3x) = 3\sin x$

$4\sin(2x+x) = 3\sin x$

$4(\sin 2x \cdot \cos x + \cos 2x \cdot \sin x) = 3\sin x$

$4(2\sin x\cos\cos x + (1-2\sin^2 x)\sin x) = 3\sin x$

$4(2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x) = 3\sin x$

$4(2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x) = 3\sin x$

$8\sin x - 8\sin^3 x + 4\sin x - 8\sin^3 x = 3\sin x$

$12\sin x - 16\sin^3 x = 3\sin x$

$-16\sin^3 x + 9\sin x = 0$

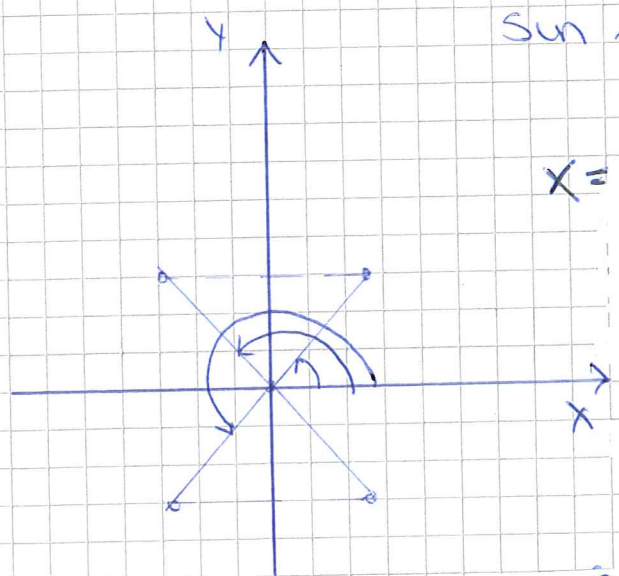
$\sin x(-16\sin^2 x + 9) = 0$

$\sin x = 0$ $\quad \underline{x = 0 \text{ eller } x = \pi}$

$-16\sin^2 x + 9 = 0$

$\sin^2 x = \frac{9}{16}$

$\sin x = \pm\sqrt{\frac{9}{16}} = \pm\frac{3}{4}$



$x = \sin^{-1}\frac{3}{4} = 0.85 \vee x = \pi - 0.85 = 2.29$

$\vee$

$x = \sin^{-1}\left(-\frac{3}{4}\right) = -0.85$, dvs

$x = -0.85 + 2\pi = 5.44$

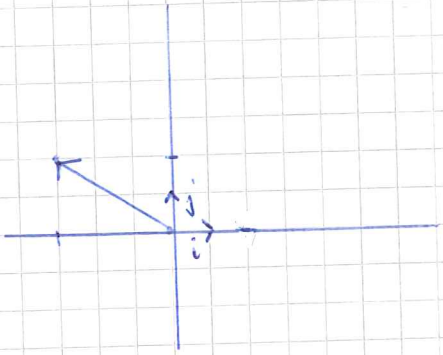
$\vee$

$x = \pi - (-0.85) = 3.99$

$\underline{x = 0 \vee x = 0.85 \vee x = 2.29 \vee x = \pi \vee x = 3.99 \vee x = 5.44}$

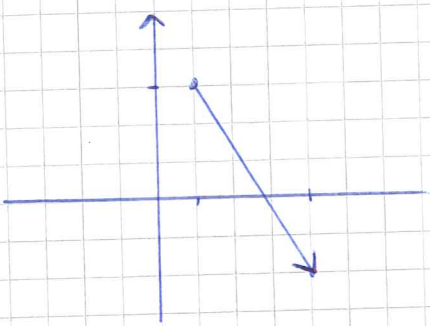
oppgave 8

a) $P_1(0,0)$ og $P_2(-3,2)$



$v = -3i + 2j$

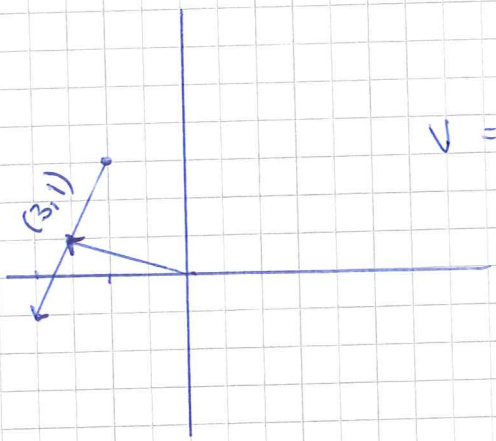
b) $P_1(1,3)$ og $P_2(4,-2)$



$v = (4-1)i + (-2-3)j$

$= 3i - 5j$

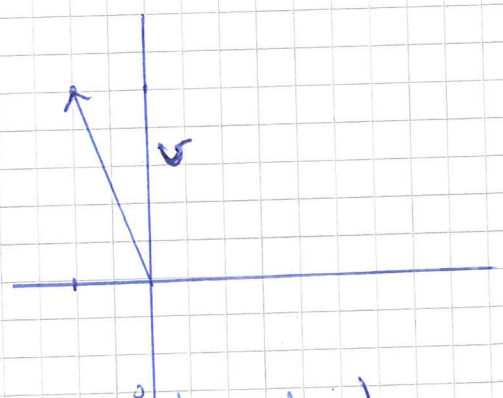
c)



$v = 3i + j$

e) $v = -2i + 5j$

$|v| = \sqrt{(-2)^2 + 25} = \sqrt{29}$



motsatt retning:

$w = 2i - 5j$

enhetsvektor: (del på lengden)

$e = \frac{2}{\sqrt{29}}i - \frac{5}{\sqrt{29}}j$

oppgave 13

(10)

$u = -3i + j$

$v = i + 4j$

$w = -2i - 3j$

a) $t = 3u + v - 2w$

$= 3(-3i + j) + (i + 4j) - 2(-2i - 3j)$

$= (-9i + 3j) + (i + 4j) + (4i + 6j)$

$= (-9 + 1 + 4)i + (3 + 4 + 6)j$

$= -4i + 13j$

b) $|t| = \sqrt{(-4)^2 + (13)^2} = \sqrt{16 + 169} = \sqrt{185}$

$\cos\theta = \frac{-4}{\sqrt{185}} = -0{,}294$

$\theta = 1{,}8693$

eller

$\theta = 4{,}4136$

$\sin\theta = \frac{13}{\sqrt{185}} = 0{,}956$

$\theta = 1{,}272$

eller

$\theta = 1{,}8693$

oppgave 18

a) $v = 2i$

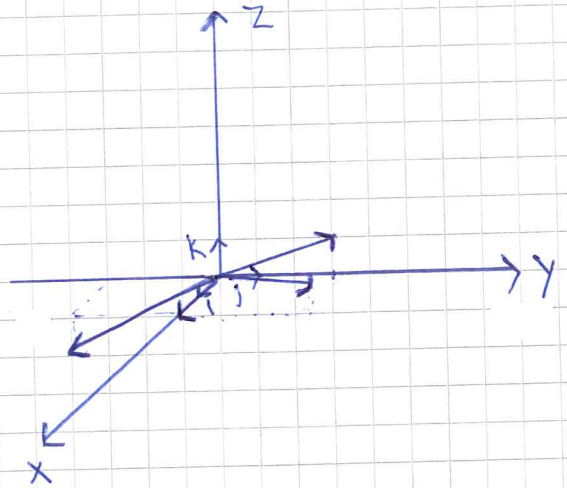
$w = 3j + k$

$k = 1$

$$v + w = (2i) + (3j + k)$$
$$= (2+0)i + (0+3)j + (0+1)k$$
$$= 2i + 3j + k$$

$$v - w = (2i) - (3j + k)$$
$$= (2-0)i + (0-3)j + (0-1)k$$
$$= 2i - 3j - k$$

$$kv = 1 \cdot v = 1 \cdot (2i) = 2i$$



Kap 4.3 oppgave 1

a) $v = i + 2j + k$

$w = 2i - j + k$

$v \cdot w = (1 \cdot 2) + (2 \cdot (-1)) + (1 \cdot 1)$

$= 2 - 2 + 1 = 1$

d) $v = 6j - 7k$

$w = 5i + 3j$

$v \cdot w = (0 \cdot 5) + (6 \cdot 3) + ((-7) \cdot 0)$

$= 18$

Oppgave 5

(14)

a) $v = i + 4j + k$

$w = 4i + tj + 3k$

Vi skal finne t som gir $v \cdot w = 0$

$$v \cdot w = (1 \cdot 4) + (4 \cdot t) + (1 \cdot 3)$$
$$= 4 + 4t + 3 = 0$$
$$4t = -7$$
$$t = -\frac{7}{4}$$

c) $v = 4i - 3k$

$w = -5j + tk$

$$v \cdot w = (4 \cdot 0) + (0 \cdot (-5)) + ((-3) \cdot t)$$
$$= 0 + 0 - 3t = 0$$
$$t = 0$$

Oppgave 14

$u = 2i + j + k$

$v = 3i - 4j + k$

$t = i - 3k$

$w = 4j + k$

Regel:

$$v \times w = (y_1 z_2 - y_2 z_1)i + (x_2 z_1 - x_1 z_2)j + (x_1 y_2 - x_2 y_1)k$$

a) $u \times v = (1 \cdot 1 - (-4) \cdot 1)i + (3 \cdot 1 - 2 \cdot 1)j + (2 \cdot (-4) - 3 \cdot 1)k$

$= 5i + j - 11k$

oppgave 15 b)

P(0,1,0)
Q(0,4,0)
R(2,0,5)

(15)

$\vec{v} = \vec{PQ} = 3j$

$\vec{w} = \vec{PR} = 2i - j + 5k$

Lengden av kryssproduktet $v \times w$ beregner arealet av parallellogrammet som v og w utspenner
Deler vi $|v \times w|$ på to finner vi arealet av trekanten.

$$v \times w = (y_1 z_2 - y_2 z_1)i + (x_2 z_1 - x_1 z_2)j + (x_1 y_2 - x_2 y_1)k$$

$$= (3 \cdot 5 - (-1) \cdot 0)i + (2 \cdot 0 - 0 \cdot 5)j + (0 \cdot (-1) - 2 \cdot 3)k$$

$$= 15i - 6k$$

$$|v \times w| = \sqrt{15^2 + 6^2} = \sqrt{261}$$

Arealet er: $\frac{\sqrt{261}}{2} = \frac{\sqrt{9 \cdot 29}}{2} = \frac{3\sqrt{29}}{2}$

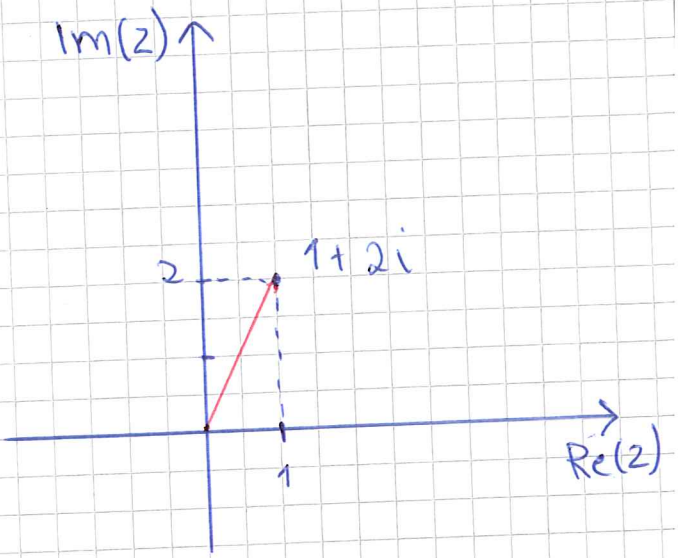
Kap 4.4

Oppgave 1

a) $1 + \sqrt{-4} = 1 + \sqrt{4 \cdot (-1)}$

$= 1 + 2\sqrt{-1}$

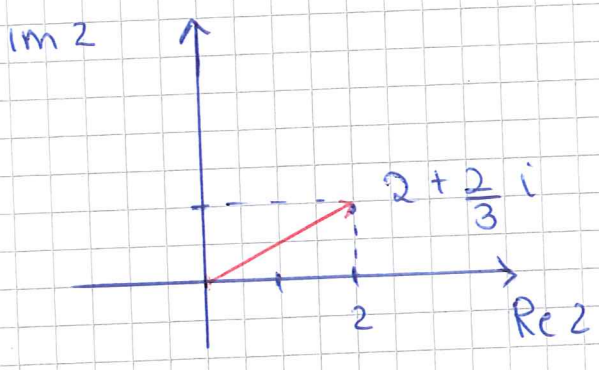
$z = 1 + 2i$



d) $2 + \sqrt{\frac{-36}{81}} = 2 + \sqrt{\frac{36}{81} \cdot (-1)}$

$= 2 + \frac{\sqrt{36}\sqrt{-1}}{\sqrt{81}} = 2 + \frac{6}{9}\sqrt{-1}$

$z = 2 + \frac{2}{3}i$



oppgave 4

(1)

a) $3 + \sqrt{-16} - 2 - 5i =$

$3 + \sqrt{16 \cdot (-1)} - 2 - 5i = 3 + 4\sqrt{-1} - 2 - 5i$

$= 3 + 4i - 2 - 5i$

$= \underline{1 - i}$

d) $\left(\sqrt{-25}\right)^2 + \sqrt{-2} \cdot \sqrt{-18} =$

Lit lang vei kanskje:

$\left(\sqrt{25 \cdot (-1)}\right)^2 + \sqrt{2 \cdot (-1)} \cdot \sqrt{18 \cdot (-1)} =$

$\left(5 \cdot \sqrt{-1}\right)^2 + \sqrt{2} \cdot \sqrt{-1} \cdot \sqrt{18}\sqrt{-1}) =$

$25 \cdot \sqrt{-1} \cdot \sqrt{-i} + \sqrt{2} \cdot \sqrt{18} \cdot \sqrt{-1} \cdot \sqrt{-1} =$

$25 \cdot \left(\sqrt{-1}\right)^2 + \sqrt{2} \cdot \sqrt{18} \cdot \left(\sqrt{-1}\right)^2 =$

$25 \cdot (-1) + \sqrt{2} \cdot \sqrt{18} \cdot (-1) =$

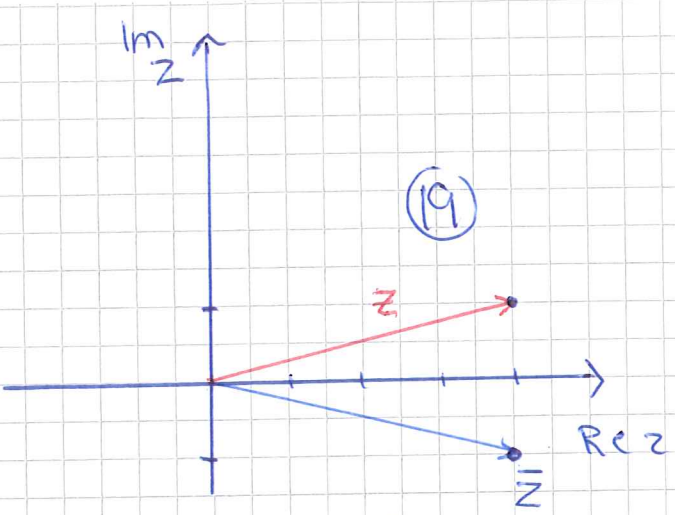
$-25 - \sqrt{2} \cdot \sqrt{9 \cdot 2} = -25 - 3\left(\sqrt{2}\right)^2$

$= -25 - 3 \cdot 2 = -25 - 6 = \underline{-31}$

oppgave 5

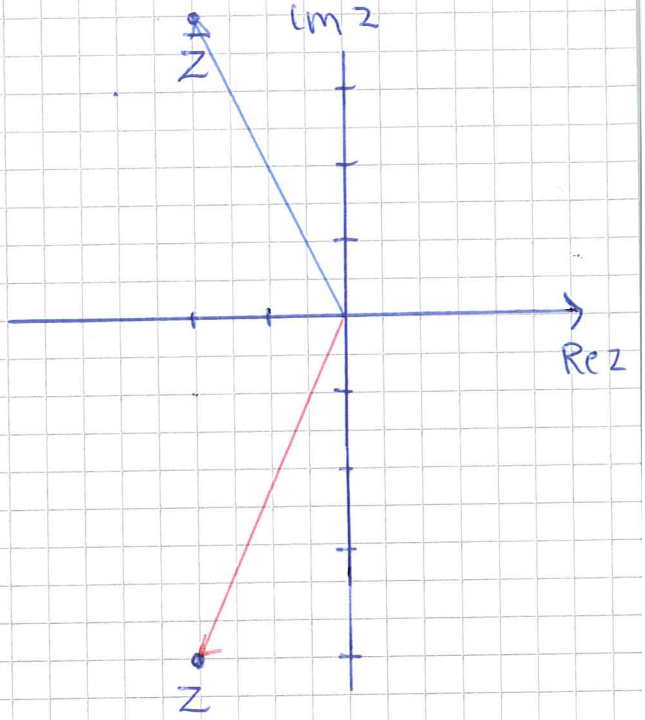
a) $z = 4 + i$

$\bar{z} = 4 - i$



d) $z = -2 - 4i$

$\bar{z} = -2 + 4i$



oppgave 6

b) $\frac{2-i}{2+i} = \frac{(2-i)(2-i)}{(2+i)(2-i)}$

: bruker den konjugerte

$= \frac{4 - 2i - 2i + i^2}{2^2 + 1^2}$

$= \frac{4 - 4i - 1}{5} = \frac{3}{5} - \frac{4}{5}i$

Άσκηση 7

(20)

a) $2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ $\quad r = 2$, $\theta = \frac{\pi}{4}$

$a = r\cos\theta = 2\cos\frac{\pi}{4} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

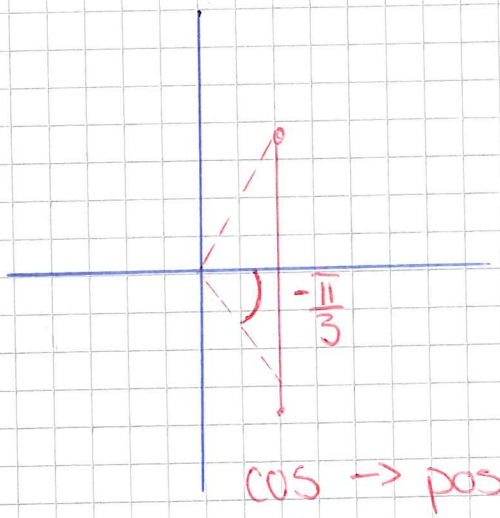
$b = r\sin\theta = 2\sin\frac{\pi}{4} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$z = \sqrt{2} + \sqrt{2}i = \underline{\sqrt{2}(1+i)}$

c) $\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)$

$a = r\cos\theta$ $\quad r = 1$

$b = r\sin\theta$ $\quad \theta = -\frac{\pi}{3}$



$a = 1\cdot\cos\left(-\frac{\pi}{3}\right) = 1\cdot\frac{1}{2} = \frac{1}{2}$

$b = 1\sin\left(-\frac{\pi}{3}\right) = 1\cdot\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{3}}{2}$

$\underline{z = \frac{1}{2} - \frac{\sqrt{3}}{2}i}$

cos → pos

sin → neg

$\cos\frac{\pi}{3} = \cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$

$\sin\left(-\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

Oppgave 8

b) $z = -\sqrt{3} + i$ $\quad a = -\sqrt{3}$, $\quad b = 1$

$$r = \sqrt{a^2 + b^2} = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$$

$\cos\theta = \frac{a}{r} = -\frac{\sqrt{3}}{2}$ $\quad \theta = \frac{5\pi}{6}$ eller $\theta = \frac{7\pi}{6}$

$\sin\theta = \frac{b}{r} = \frac{1}{2}$ $\quad \theta = \frac{\pi}{6}$ eller $\theta = \frac{5\pi}{6}$

$\theta = \frac{5\pi}{6}$

$$\underline{z = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)}$$

Oppgave 10

a) $e^{i\left(\frac{\pi}{3}\right)} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

$r = 1 \quad \theta = \frac{\pi}{3}$

$\cos\theta = \frac{a}{r} \qquad a = \cos\theta \cdot r$

$= \cos\frac{\pi}{3} \cdot 1 = \frac{1}{2}$

$\sin\theta = \frac{b}{r} \qquad b = \sin\theta \cdot r$

$\sin\frac{\pi}{3} \cdot 1 = \frac{\sqrt{3}}{2}$

$z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

c) $e^{i\left(-\frac{\pi}{6}\right)} = \cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)$

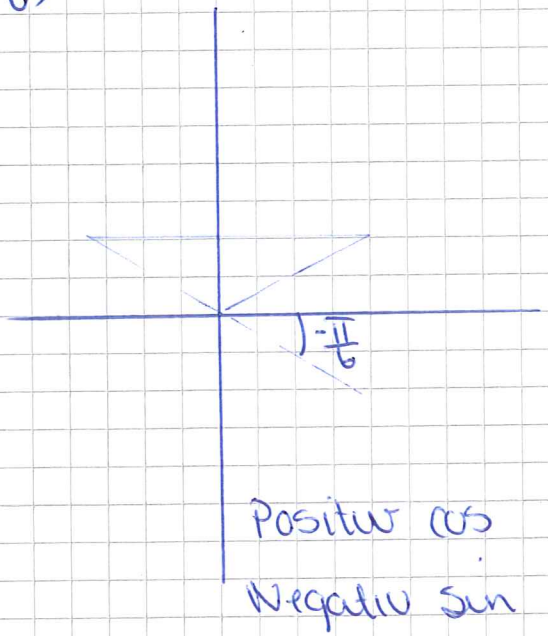
$a = \cos\theta \cdot r$

$= \cos\left(-\frac{\pi}{6}\right) \cdot 1 = \frac{\sqrt{3}}{2}$

$b = \sin\theta \cdot r$

$= \sin\left(-\frac{\pi}{6}\right) \cdot 1 = -\frac{1}{2}$

$z = \frac{\sqrt{3}}{2} - \frac{1}{2}i$



$\cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$

Oppgave 11

c) $1 + \sqrt{3}\,i$ $\qquad a = 1$, $\quad b = \sqrt{3}$

$$r = \sqrt{1^2 + \sqrt{3}^2} = \sqrt{4} = 2$$

$$\cos\theta = \frac{a}{r} = \frac{1}{2} \qquad \theta = \frac{\pi}{3} \text{ eller } \frac{5\pi}{3}$$

$$\sin\theta = \frac{b}{r} = \frac{\sqrt{3}}{2} \qquad \theta = \frac{\pi}{3} \text{ eller } \frac{2\pi}{3}$$

$$z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2e^{i\left(\frac{\pi}{3}\right)}$$