Oppgaver 10.1

Oppgave 1b

$$3x_1 - 2x_2 + 5x_3 = 0$$
$$8x_2 + 3x_3 = -5$$
$$x_1 = 0$$

Koeff. matrise: $\begin{bmatrix} 3 & -2 & 5 \\ 0 & 8 & 3 \\ 1 & 0 & 0 \end{bmatrix}$

Totalmatrise: $\left[\begin{array}{ccc|c} 3 & -2 & 5 & 0 \\ 0 & 8 & 3 & -5 \\ 1 & 0 & 0 & 0 \end{array}\right]$

Oppgave 2

a) trappeform, men ikke redusert trappeform fordi hver kolonne som innenolder et ledende 1-tall ikke har bare nuller ellers.

b) redusert trappeform

c) Ingen av delene, en rekke som bare består av nuller skal samles i bunnen for å være på trappeform.

d) redusert trappeform

e) trappeform (samme årsak som i a)

Oppgaver Kap 10.2

oppgave 1b

$$\begin{bmatrix} 6 & 0 \\ -3 & -1 \\ 5 & -4 \end{bmatrix}$$

antall rekker $(m) = 3$

antall kolonner $(n) = 2$

3×2 matrise

$a_{11} = 6$ $\quad a_{12} = 0$

$a_{21} = -3$ $\quad a_{22} = -1$

$a_{31} = 5$ $\quad a_{32} = -4$

oppgave 2

a) kvadratisk og triangulær

b) Ingen av formene

c) diagonal og kvadratisk

d) kvadratisk og triangulær

oppgave 3 b

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$A = A^T$$

$$A^T = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

c) $A = \begin{bmatrix} 4 & 2 & 0 \\ -2 & 5 & -1 \\ -1 & 0 & 2 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 4 & -2 & -1 \\ 2 & 5 & 0 \\ 0 & -1 & 2 \end{bmatrix}$

oppgave 4

$$A = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -2 \\ 5 & 1 \end{bmatrix}$$

a) $A + B = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -2 \\ 4 & 4 \end{bmatrix}$

d) $2B - A = 2\begin{bmatrix} 4 & -2 \\ 5 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 8 & -4 \\ 10 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 6 & -4 \\ 11 & -1 \end{bmatrix}$$

e) $(A - B)^T = \left(\begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 5 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} -2 & 2 \\ -6 & 2 \end{bmatrix}^T$

$$= \begin{bmatrix} -2 & -6 \\ 2 & 2 \end{bmatrix}$$

oppgave 6

$A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix}$ $C = \begin{bmatrix} -3 & 1 \\ 2 & 4 \end{bmatrix}$

a) $A \cdot B = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 10 & -5 \end{bmatrix}$

c) $A^2 = A \cdot A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 5 & 9 \end{bmatrix}$

e) $B(A+C) = BA + BC$

$= \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} 0 & -6 \\ 5 & -3 \end{bmatrix} + \begin{bmatrix} -7 & -7 \\ -11 & -1 \end{bmatrix} = \begin{bmatrix} -7 & -13 \\ -6 & -4 \end{bmatrix}$

oppgave 11

a) $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} = \frac{1}{6-5} \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

c) $$A = \left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 \end{array}\right] 1 = \left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 & 1 \end{array}\right] 2 =$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 & 1 \end{array}\right] -1 = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 & 1 \end{array}\right] \times (-1)$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix}$$

oppgave 12 a)

$$A = \begin{bmatrix} 2 & -3 \\ 4 & -1 \end{bmatrix} \quad b = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$Ax = b$$
$$x = A^{-1} b$$

$$\det A = 2 \cdot (-1) - 4 \cdot (-3) = -2 + 12 = 10$$

$$A^{-1} = \frac{1}{10} \begin{bmatrix} -1 & 3 \\ -4 & 2 \end{bmatrix}$$

$$x = \frac{1}{10} \begin{bmatrix} -1 & 3 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 13 \\ 12 \end{bmatrix} = \begin{bmatrix} 13/10 \\ 12/10 \end{bmatrix}$$

Oppgaver Kap 10.3

Oppgave 1 a)

$$A = \begin{bmatrix} 3 & 2 \\ -1 & 3 \end{bmatrix}$$

$$\det A = 3 \cdot 3 - 2 \cdot (-1) = 9 + 2 = \underline{11}$$

Oppgave 1 d)

$$A = \begin{bmatrix} 1 & 3 & -4 \\ 0 & -7 & 5 \\ 0 & 0 & -2 \end{bmatrix}$$

$$\det A = a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$$

$$A_{11} = (-1)^2 \det M_{11} = \det \begin{bmatrix} -7 & 5 \\ 0 & -2 \end{bmatrix} = (-7) \cdot (-2) - 0 = 14$$

$$A_{12} = (-1)^3 \det M_{12} = (-1) \det \begin{bmatrix} 0 & 5 \\ 0 & -2 \end{bmatrix} = (-1) \cdot 0 \cdot 0 = 0$$

$$A_{13} = (-1)^4 \det M_{13} = \det \begin{bmatrix} 0 & -7 \\ 0 & 0 \end{bmatrix} = 0$$

$$\det A = 1 \cdot 14 = \underline{14}$$

eller siden dette er en trappematrise

$$\det A = 1 \cdot (-7) \cdot (-2) = \underline{14}$$

Oppgave 2 a)

$$A = \begin{bmatrix} t & 1 \\ 3 & t-2 \end{bmatrix}$$

$$\det A = t(t-2) - 3 \cdot 1$$

$$= \underline{t^2 - 2t - 3}$$

↓ faktorisere

$$t^2 - 2t - 3 = 0$$

$$t = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1}$$

$$= \frac{2 \pm \sqrt{4 + 12}}{2}$$

$$= \frac{2 \pm 4}{2}$$

$$t_1 = \frac{2+4}{2} = 3$$

$$t_2 = \frac{2-4}{2} = -1$$

$$\det A = (t-3)(t-(-1)) = \underline{(t-3)(t+1)}$$

oppgave 9c

$$A = \begin{array}{ccccc} & & - & - & - \\ \left[\begin{array}{ccc} 3 & 1 & -2 \\ -2 & 3 & 7 \\ 4 & 5 & 3 \end{array}\right. & \left|\begin{array}{cc} 3 & 1 \\ -2 & 3 \\ 4 & 5 \end{array}\right] \\ & & + & + & + \end{array}$$

$$\det A = 27 + 28 + 20 + 24 - 105 + 6 = 0$$

$\supset$: Singulær

kap 10.6 oppgave 4

Vi skal bestemme den krypterte meldingen på matriseform når:

a) $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

$$M = \begin{bmatrix} S & V \\ A & R \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 1 & 18 \end{bmatrix}$$

$$C = M \cdot A = \underbrace{\begin{bmatrix} 19 & 22 \\ 1 & 18 \end{bmatrix}}_{2\times2} \underbrace{\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}}_{2\times2} = \underbrace{\begin{bmatrix} 60 & 161 \\ 20 & 59 \end{bmatrix}}_{2\times2}$$

b) $A = \begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}$

$$M = \begin{bmatrix} P & Å \\ F & E \\ R & I \\ E & \end{bmatrix} = \begin{bmatrix} 16 & 29 \\ 6 & 5 \\ 18 & 9 \\ 5 & 0 \end{bmatrix}$$

$$C = MA = \underbrace{\begin{bmatrix} 16 & 29 \\ 6 & 5 \\ 18 & 9 \\ 5 & 0 \end{bmatrix}}_{4\times2} \underbrace{\begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}}_{2\times2} = \underbrace{\begin{bmatrix} 151 & -106 \\ 39 & -28 \\ 99 & -72 \\ 20 & -15 \end{bmatrix}}_{4\times2}$$

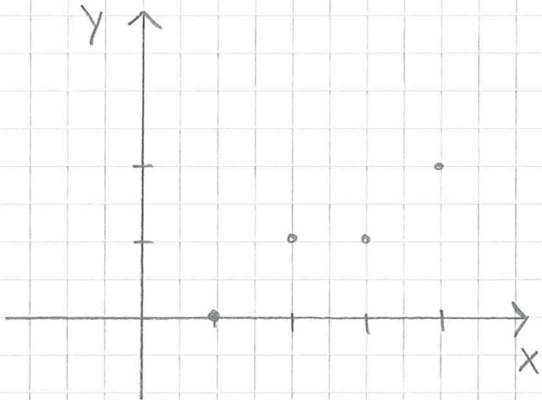
Delkapittel 10.8:

Oppgave 1

Vi skal bestemme den rette linjen $y = a_0 + a_1 x$ som er tilpasset de gitte punktene (x, y)

Vi benytter minste kvadraters metode

a) $(1,0), (2,1), (3,1), (4,2)$



Vi får følgende matriser:

$$x = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \quad y = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

$$A^T A x = A^T y$$

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}}_{2 \times 4} \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}}_{4 \times 2} \underbrace{\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}}_{2 \times 1} = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}}_{2 \times 4} \underbrace{\begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}}_{4 \times 1}$$

$$\begin{bmatrix} 4 & 10 \\ 10 & 30 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 4 & 10 & 4 \\ 10 & 30 & 13 \end{array}\right] \times \left(\tfrac{1}{4}\right) = \left[\begin{array}{cc|c} 1 & 5/2 & 1 \\ 10 & 30 & 13 \end{array}\right] \; -10 = \left[\begin{array}{cc|c} 1 & 5/2 & 1 \\ 0 & 5 & 3 \end{array}\right] \times \left(\tfrac{1}{5}\right)$$

$$\left[\begin{array}{cc|c} 1 & 5/2 & 1 \\ 0 & 1 & 3/5 \end{array}\right] \; -\tfrac{5}{2} = \left[\begin{array}{cc|c} 1 & 0 & -1/2 \\ 0 & 1 & 3/5 \end{array}\right]$$

$$a_0 = -\frac{1}{2}$$

$$a_1 = \frac{3}{5}$$

$$\underline{\underline{y = -\frac{1}{2} + \frac{3}{5} x}}$$

Oppgave 1 Kap. 10.8

b) (0, 3.2), (1, 2.2), (2, 1.9), (3, 1.3)

Vi får følgende matriser:

$$X = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \quad Y = \begin{bmatrix} 3.2 \\ 2.2 \\ 1.9 \\ 1.3 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$$A^T A x = A^T Y$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3.2 \\ 2.2 \\ 1.9 \\ 1.3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 6 \\ 6 & 14 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 8{,}6 \\ 9{,}9 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 4 & 6 & 8{,}6 \\ 6 & 14 & 9{,}9 \end{array}\right] \times (\tfrac{1}{4}) = \left[\begin{array}{cc|c} 1 & 3/2 & 2{,}15 \\ 6 & 14 & 9{,}9 \end{array}\right] (-6) = \left[\begin{array}{cc|c} 1 & 3/2 & 2{,}15 \\ 0 & 5 & -3 \end{array}\right] \times (\tfrac{1}{5})$$

$$\left[\begin{array}{cc|c} 1 & 3/2 & 2{,}15 \\ 0 & 1 & -3/5 \end{array}\right] (-\tfrac{3}{2}) = \left[\begin{array}{cc|c} 1 & 0 & 3{,}05 \\ 0 & 1 & -0{,}6 \end{array}\right]$$

$a_0 = 3{,}05$

$a_1 = -0{,}6$

$$\underline{\underline{y = 3{,}05 - 0{,}6x}}$$