EKSAMEN - MATEMATIKK 1 - HØST 2012

Oppgave 1

$$\cos(2x) - 5\sin x = 3$$

$$1 - 2\sin^2 x - 5\sin x - 3 = 0$$

$$-2(\sin x)^2 - 5\sin x - 2 = 0$$

$$y = \sin x$$

$$-2y^2 - 5y - 2 = 0$$

$$y = \frac{-(-5) \pm \sqrt{(-5)^2 - 4\cdot(-2)\cdot(-2)}}{2\cdot(-2)}$$

$$y = \frac{5 \pm \sqrt{25-16}}{-4}$$

$$y = \frac{5 \pm 3}{-4}$$

$$y_1 = \frac{5+3}{-4} = \frac{8}{-4} = -2$$

$$y_2 = \frac{5-3}{-4} = -\frac{2}{4} = -\frac{1}{2}$$

$\sin x = -2$ (eksister ikke)

$\sin x = -\frac{1}{2}$ gir $\underline{\underline{x = \frac{7\pi}{6} \text{ og } x = \frac{11\pi}{6}}}$

Oppgave 2

$v = 3i + 2j + k$
$w = -2i + 4j + tk$

a) $v \cdot w = (x_1 x_2 + y_1 y_2 + z_1 z_2)$
$= (3 \cdot (-2) + 2 \cdot 4 + 1 \cdot t) = 0$
$= -6 + 8 + t = 0$
$t = -8 + 6$
$\underline{\underline{t = -2}}$

b) $v \times w = (\underset{x_1}{3i} + \underset{y_1}{2j} + \underset{z_1}{k}) \times (\underset{x_2}{-2i} + \underset{y_2}{4j} \underset{z_2}{-2k})$

$= (y_1 z_2 - y_2 z_1)i + (x_2 z_1 - x_1 z_2)j + (x_1 y_2 - x_2 y_1)k$
$= (2 \cdot (-2) - 4 \cdot 1)i + (-2 \cdot 1 - 3 \cdot (-2))j + (3 \cdot 4 - (-2) \cdot 2)k$
$= (-4 - 4)i + (-2 + 6)j + (12 + 4)k$
$= \underline{\underline{-8i + 4j + 16k}}$

Dersom $t = 3$

$v \times w = (\underset{x_1}{3i} + \underset{y_1}{2j} + \underset{z_1}{k}) \times (\underset{x_2}{-2i} + \underset{y_2}{4j} + \underset{z_2}{3k})$

$= (2 \cdot 3 - 4 \cdot 1)i + ((-2) \cdot 1 - 3 \cdot 3)j + (3 \cdot 4 - (-2) \cdot 2)k$
$= \underline{2i - 11j + 16k}$

Oppgave 3

Polarform: $z = 2\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$

$r = 2$

$\theta = \frac{\pi}{4}$

Vet at

$\cos\theta = \frac{x}{r} \rightarrow x = r\cos\theta$

$= 2\cos\left(\frac{\pi}{4}\right) = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$

$\sin\theta = \frac{y}{r} \rightarrow y = r\sin\theta$

$= 2\sin\left(\frac{\pi}{4}\right) = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$

Kartesisk form:

$\underline{z = \sqrt{2} + \sqrt{2}\, i}$

Eksponentialform:

$\underline{z = 2e^{i\frac{\pi}{4}}}$

Oppgave 4

a)
$$\lim_{x \to 2} \frac{x-2}{x^2+x-6}$$

$$= \lim_{x \to 2} \frac{x-2}{(x-2)(x+3)} = \lim_{x \to 2} \frac{1}{x+3} = \frac{1}{5}$$

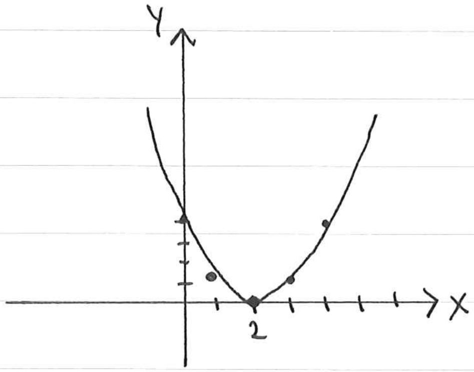
eller L'Hopitals regel

$$\lim_{x \to 2} \frac{x-2}{x^2+x-6} = \lim_{x \to 2} \frac{1}{2x+1} = \frac{1}{5}$$

oppgave 4

b)

$$y = f(x) = x^2 - 4x + 4$$



$x \geq 2$

$y \geq 0$

- $f(x)$ er strengt voksende for $x \geq 2$, dvs en-entydig og har derfor en invers funksjon

Trinn 1: $y = f(x) = x^2 - 4x + 4$

$$y = (x-2)^2$$

$$\pm\sqrt{y} = x - 2$$

$$x = \pm\sqrt{y} + 2$$

Siden $x \geq 2$ må y være positiv

$$= \sqrt{y} + 2$$

Trinn 2: $f^{-1}(y) = \sqrt{y} + 2$

Trinn 3: $\underline{f^{-1}(x) = \sqrt{x} + 2 \qquad x \geq 0}$

Oppgave 5

$y = f(x) = e^{x^2}$

$p(1, e)$

$a \quad f(a)$

$f'(x) = e^{x^2} \cdot 2x$

$= 2x e^{x^2}$

$f'(a) = f'(1) = 2 \cdot e^1 = 2e$

Likningen for tangenten:

$y = f'(a)(x-a) + f(a)$

$y = 2e(x-1) + e$

$= 2ex - 2e + e$

$= 2ex - e$

$= \underline{e(2x-1)}$

oppgave 6

$$x^2 - 3xy + e^x y = 2$$

$$2x - \left(3y + 3x\frac{dy}{dx}\right) + \left(e^x y + e^x \frac{dy}{dx}\right) = 0$$

$$2x - 3y - 3x\frac{dy}{dx} + e^x y + e^x \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(e^x - 3x\right) = 3y - 2x - e^x y$$

$$\frac{dy}{dx} = \frac{3y - 2x - e^x y}{e^x - 3x}$$

oppgave 7

$$y = f(x) = \frac{2-x^2}{x^2-1} \qquad x \neq \pm 1$$

1) Skjæring med koordinataksene:

X-aksen: $y=0$

$$\frac{2-x^2}{x^2-1} = 0$$

$$2 - x^2 = 0$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

⊃: Skjærer x-aksen i $(\sqrt{2},0)$ og $(-\sqrt{2},0)$

Y-aksen: $X=0$

$$y = \frac{2-0^2}{0^2-1} = -2$$

⊃: skjærer y-aksen i $(0,-2)$

2) Symmetri:

$$f(-x) = \frac{2-(-x)^2}{(-x)^2-1} = \frac{2-x^2}{x^2-1} = f(x)$$

⊃: Symmetri om y-aksen

3) Asymptoter

Vertikale asymptoter finner vi der neuneren ikke er definert:

$X = 1$ og $X = -1$

$$
\begin{array}{lll}
x \to 1^+ & f(x) \to \infty & \\
x \to 1^- & f(x) \to -\infty & \\
\end{array}
\Bigg\}\ \begin{array}{l} X = 1 \\ \text{vertikal asymp.} \end{array}
$$

$$
\begin{array}{lll}
x \to -1^+ & f(x) \to -\infty & \\
x \to -1^- & f(x) \to \infty & \\
\end{array}
\Bigg\}\ \begin{array}{l} X = -1 \\ \text{er vertikal asymp.} \end{array}
$$

Horisontale asymptoter:

$$
\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{2 - x^2}{x^2 - 1} = \lim_{x \to \infty} \frac{\frac{2}{x^2} - \frac{x^2}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}}
$$

$$
= \lim_{x \to \infty} \frac{\overset{\to 0}{\frac{2}{x^2}} - 1}{1 - \underset{\to 0}{\frac{1}{x^2}}} = \frac{-1}{1} = -1
$$

$y = 1$ er horisontal asymptote

Ingen skråasymp.

4) Ekstremalverdier

$$f'(x) = \frac{-2x\,(x^2-1) - (2-x^2)\cdot 2x}{(x^2-1)^2}$$

$$= \frac{-2x\,(x^2-1+2-x^2)}{(x^2-1)^2}$$

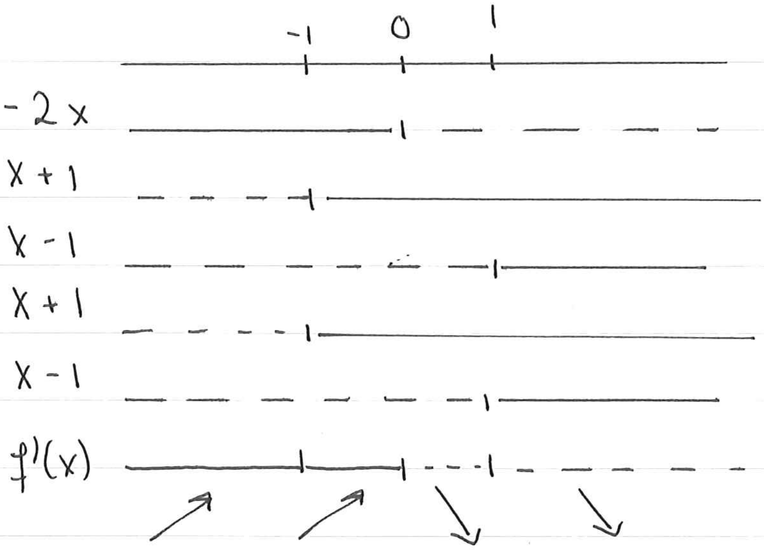
$$= \frac{-2x}{(x^2-1)^2}$$

$$f'(x) = 0$$

$$\frac{-2x}{(x^2-1)^2} = 0$$

$$-2x = 0$$

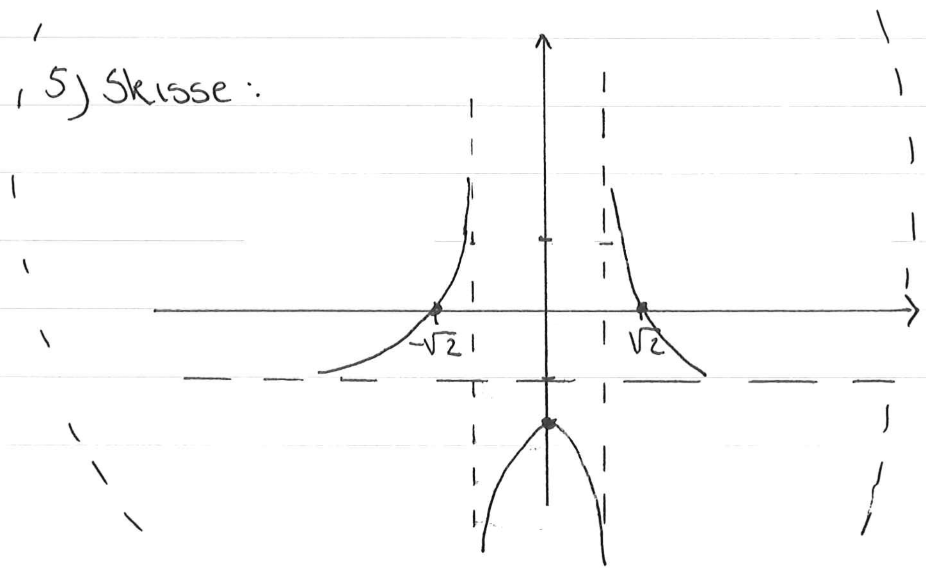
$$\underline{x = 0}$$



$x = 0$ gir lokalt max

$f(0) = -2$

5) Skisse:



Oppgave 8

$a(t) = \sqrt{2t}$ $\quad V_0 = V(0) =$

$$V(t) = \int a(t)\,dt = \int \sqrt{2t}\,dt$$

$u(t) = 2t$

$\frac{du}{dt} = 2$

$du = 2dt$

$dt = \frac{1}{2} du$

$$= \int \sqrt{u}\,\frac{1}{2}\,du = \frac{1}{2}\int u^{\frac{1}{2}}\,du$$

$$= \frac{1}{2}\,\frac{1}{1+\frac{1}{2}}\,u^{\frac{1}{2}+1} + C$$

$$= \frac{1}{2}\,\frac{2}{3}\,u^{3/2} + C$$

$$= \frac{1}{3}u^{\frac{3}{2}} + C$$

$$= \frac{1}{3}(2t)^{\frac{3}{2}} + C$$

$$V(0) = \frac{1}{3}(2 \cdot 0)^{\frac{3}{2}} + C = 1$$

$$C = 1$$

$$V(t) = \frac{1}{3}(2t)^{\frac{3}{2}} + 1$$

oppgave 9

$$\int \frac{x^2+5x-6}{x^2-2x}\,dx$$

(1) Divisjon / Polynomdivisjon:

$$x^2+5x-6 : x^2-2x = 1 + \frac{7x-6}{x^2-2x}$$
$$x^2-2x$$
$$7x-6$$

(2) Faktorisering av nevner:

$$x^2-2x = x(x-2)$$

(3) Delbrøkoppspaltn.

$$\frac{7x-6}{x^2-2x} = \frac{7x-6}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}$$

$$\frac{(7x-6)\,x\cancel{(x-2)}}{x\cancel{(x-2)}} = \frac{A\cancel{x}(x-2)}{\cancel{x}} + \frac{Bx\cancel{(x-2)}}{\cancel{x-2}}$$

$$7x-6 = A(x-2) + Bx$$

$x=0$ gir $\quad 7\cdot 0 - 6 = A\cdot(-2) + B\cdot 0$

$$-6 = -2A$$

$$2A = 6 \qquad A = \frac{6}{2} = 3$$

$x=2$ gir $\quad 7\cdot 2 - 6 = A(2-2) + 2B$

$$8 = 2B$$

$$B = \frac{8}{2} = 4$$

$$\int \frac{x^2+5x-6}{x^2-2x}\,dx = \int\left(1+\frac{3}{x}+\frac{4}{x-2}\right)dx$$

$$= \int 1\,dx + 3\int \frac{1}{x}\,dx + 4\int \frac{1}{x-2}\,dx$$

$$= x + 3\ln|x| + 4\ln|x-2| + C$$